QUIZ 2: INTRODUCTION TO ALGEBRAIC GEOMETRY

Date: **March 10th 2021** Duration: 1 hr 30 minutes
A **ring** would mean a **commutative ring with identity**.

(1) (10 points) Let A be a normal domain (i.e. integrally closed domain). Let B be an integral domain contain A as a subring and $\alpha \in B$ be integral over A. Let K be the field of fractions of A and assume that $K(\alpha)/K$ is a separable extension. Show that the minimal polynomial of α over K have coefficients in A.

(2) (10 points) Let k be an algebraically closed field. Consider the affine algebraic set $X(n)$ in $\mathbb{A}_k^2$ defined by the polynomial $y^n - x^2 \in k[x, y]$ where n is a fixed positive integer. Show that $X(n)$ is a variety if n is odd. Show that $X(1)$ and $X(3)$ are homeomorphic.